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## E-mobility ID-codes

The purpose of IDs, ID usage and ID format

## The purpose and usage of e-mobility IDs

E-mobility ID codes are IDs with a national country code for Mobility Service Providers (MSP) and Charge Station Owners (CSO) or Charge Point Operators (CPO). These unique IDs for organizations that manage charge stations or offer charge services to EV drivers are needed to identify these organizations for international billing and data exchange. Issuing and managing codes for EV driver contracts and charging stations ensures that charging stations can be found throughout Europe andthat transactions for access and payment are reliable.

The code consist of two parts: the first part to identify the CSO/CPO and MSP and the second part to identify the individual recharging point and contract within these organizations.
The IDRO issues only the first 5 digits of - IDs for CSO/CPOs and MSPs, and the CSO/CPO and MSP issue the second part to identify respectively the specific charge point (EVSE) or contract (EMA). Forthe CSO/CPO this total ID of part one and two is called EVSE-ID (Electric Vehicle Supply Equipment) and for the MSP this is called the EMA-ID (Electric Mobility Account).They are conceived by CSO/CPOs and MSPs purely as identifiers. Consequently, they are not meant to contain other information and should not be considered as marketing tools.

## ID format part 1 (first 5 characters) and 2 (remaining characters)

The IDACS Consortium agreed to use the current format for part one and two as this is adequate for European e-mobility ID issuing and usable for the market. This format was initially published by ISO as part of ISO 15118 and eMI3. eMI3 specified and clarified some characters of the ISO 15118-2:2014code.

For the use of the ID-format, the IDACS Consortium:

- strongly advises companies NOT to use the optional separators between IT systems. They aremeant for visibility only, not for IT communication. It is up to individual companies how to display the ID's and where which separators are put.
- leaves it up to Mobility Service Provider (MSP) to use or not use the 'Check digit', as it is mainly for their own benefits and usage and it has no impact on connected organisations, like CPO's.
- requires the "Type character' to be used in all new situations for Contracts with "C" as 'type character', and at least an EVSE ID is needed for all charge points with "E" as 'type character'.If the CPO or CSO is also using the ID's for Pools or Stations is up to the CSO/CPO. If used thatway a "P" or " S " must be used. It is meant for use cases addressing POI on maps, such as request for reservation of an EVSE.

The IDACS Consortium acknowledges that possible changes on the format in the future can beprocessed based on consensus.

## Co-financed by the European Union

Connecting Europe Facility
$\begin{array}{ll} & \text { e-Mobility Account Identifier } \\ \text { EMAID } & \text { For Mobility Service Provider }\end{array}$

| Issued by | ID Registration Organisations IDRO |  |  | Emobility Provider |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Descriptio <br> n | Country | Seperator | EMP | Seperator | Type* | Contract ID instance | Seperator | Check digit |
| Example | DE | "-" | AB1 | "-" | C | 12A23GHI | "-" | K |
| Explanati on | 2 characters <br> (alphabetic) <br> [ISO 3166-1 <br> Alpha-2] \{2\} | optional <br> [-] \{1\} | 3 characters (alphanumeric) [A-Z;0-9]\{3\} | optional $[-]\{1\}$ | 1 character type identifier (alphabetic) [C]\{1\} | 8 characters (alphanumeric) [A-Z;a-z;0-9]\{8\} | optional <br> [-] \{1\} | optional calculated check digit $\begin{gathered} {[A-Z ; 0-} \\ 9]\{1\} \end{gathered}$ |

*) The 'Type character' should be used in all new situations for Contracts with " $C$ " as 'type character', and at least an EVSE ID is needed for all charge points with "E" as 'type character'. If the CPO or CSO is also using the IDs for Pools or Stations is up to the CSO/CPO. If used that way a " P " or " S " must be used. It is meant for use cases addressing POI on maps, such as request for reservation of an EVSE.

## Electronic Vehicle Support <br> EVSE ID Equipment ID <br> For Charge Point Operator

| Issued by | ID Registration Organisations IDRO |  |  | Charge point operator/unit |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\begin{gathered} \text { Descriptio } \\ \mathrm{n} \end{gathered}$ | Country | Seperator | EMP | Seperator | Type* | charge point ID |
| Example | DE | "*" | AB1 | "*" | E | 2542AX8769 |
| Explanati on | 2 characters <br> (alphabetic) <br> [ISO 3166-1 <br> Alpha-2] \{2\} | optional <br> [*] \{1\} | 3 characters (alphanumeric) [A-Z;0-9]\{3\} | optional <br> [*] \{1\} | 1 character type identifier (alphabetic) [E, P or S]\{1\} | up to 30 characters (alphanumeric) <br> [A-Z;a-z;0-9]\{max 30\} |

Figure 1: Updated agreed ID formats
(version 2023)

Annex 1 presents the check digit calculation for contract-IDs
Source: OCHP (Version 05.03.2014) - https://www.ochp.eu/wp-content/uploads/2014/02/E-
Mobility-IDs EVCOID Check-Digit-Calculation Explanation.pdf

## Annex 1: Check Digit Calculation for Contract-IDs

## Check Digit Calculation for Contract-IDs

Introduction ..... 1
Mathematical Algorithm ..... 2
Initiation for the Contract-ID ..... 2
Setup for the Contract-ID ..... 2
Lookup Tables ..... 4
Example ..... 5
Implementation ..... 6
Acknowledgment ..... 8

## Introduction

The Contract Identifier (short: CID; also known as eMA-ID or EVCO-ID) as described by theeMI ${ }^{3}$ Group and standardized in ISO/IEC-15118 Annex H allows specifying an optional but highly recommended check digit. The purpose of the check digit is the detection of typing errors in human-machine interaction. The syntax of a Contract-ID is:

```
<CID> = <Country Code> <S> <Provider ID> <S> <ID Type> <ID Instance> <S> <Check Digit>
```

This syntax is based on DIN SPEC 91286 (2011), from where ISO/IEC-15118 adapts and extends it for further international use. The in there specified former check digit is not empowered to detect all common typing errors. Therefore, the here described new algorithmwas introduced with the novel of the Contract-ID since it performs better than existing systems such as ISO/IEC 7064, MOD 37, 36 and ISO/IEC 7064 1271-36

The check digit system described within this document can detect the five most frequent errortypes made by human operators transmitting a character sequence:
$1)$ single error: $\quad \cdots \cdots a \cdots \cdots \cdot \quad \rightarrow \cdots b \cdots \cdot$
2) adjacent transposition: $\cdots \cdots \cdot \mathrm{ab} \cdots \cdot \rightarrow \quad \cdots \cdot \cdot \mathrm{ba} \cdots$
3) twin error: $\quad \cdots \cdot \cdot \mathrm{aa} \cdots \cdot \rightarrow \quad \cdots \cdot \mathrm{bb} \cdots$
4) jump transposition: $\cdots \cdot \cdot \mathrm{abc} \cdots \quad \rightarrow \quad \cdots \cdot \mathrm{cba} \cdots$
5) jump twin error: $\cdots \cdots a c a \cdots \quad \rightarrow \quad \cdots \cdot{ }^{\cdots} \cdot \overrightarrow{ } \cdot \cdots$

The mathematical theory behind is explained by Chen et al in the article:
Chen, Y., Niemenmaa, M., \& Vinck, A. (2013). A check digit system over a group of arbitrary order. 2013 8th International Conference on Communications and Networking in China (CHINACOM) (pp. 897-902). IEEE. http://ieeexplore.ieee.org/xpl/articleDetails.jsp?arnumber=6694722

## Mathematical Algorithm

Over the 36 alpha-numeric characters, results in the group $\left(Z_{2} \times Z_{2}\right) \times\left(Z_{3} \times Z_{3}\right)$ are to be applied. $Z_{2}$ is the modulo group 'mod 2' and contains only 0 and 1 as values, i.e. all even values correspond to 0 and all odd values to 1 . Correspondingly, $Z_{3}$ is the modulo group 'mod 3' and contains only 0,1 and 2 as values, i.e. 3 corresponds to 0,4 to 1,5 to 2,6 to 0 ,etc. The steps to calculate the check digit in the theory are:

1. For each $a$ from $0,1, \ldots 35$ there exist unique $q$ and $r$ such that $a=q \cdot 9+r$, where $q$ can be considered as an element in $\left(Z_{2} \times Z_{2}\right)$; and $r$ as an element in $\left(Z_{3} \times Z_{3}\right)$.
Thus, for a string with $n$ characters, $a_{1}, \ldots, a_{n}$, we easily have ( $q_{1}, \ldots, q_{n}$ ) and ( $r_{1}, \ldots$, $r_{n}$ ), where $q_{i}$ and $r_{i}$ are the quotient and remainder, respectively when dividing $a_{i}$ by 9. (In the check equation, $\mathrm{q}_{\mathrm{i}}$ and $\mathrm{r}_{\mathrm{i}}$ are considered as elements of $\left(\mathrm{Z}_{2} \times \mathrm{Z}_{2}\right)$ and $\left(\mathrm{Z}_{3} \times\right.$ $\mathrm{Z}_{3}$ ), respectively.)
2. To calculate the check digit $a_{n+1}$, two check equations are used. In particular two matrices are used: the binary matrix $\mathrm{P}_{1}$ and the other ternary matrix $\mathrm{P}_{2}$, where $\mathrm{P}_{1}$ is a matrix which has $x^{2}+x+1$ as its characteristic polynomial; and $P_{2}$ is a matrix whosecharacteristic matrix is $x^{2}+x+2$.
3. Then we can calculate $\mathrm{q}_{\mathrm{n}+1}$ and $\mathrm{r}_{\mathrm{n}+1}$ from the following two check equations, respectively.
$\mathrm{q}_{1} \mathrm{P}_{1}+\mathrm{q}_{2} \mathrm{P}_{1}^{2}+\ldots+\mathrm{q}_{\mathrm{n}} \mathrm{P}_{1}^{\mathrm{n}}+\mathrm{q}_{\mathrm{n}+1} \mathrm{P}_{1}^{\mathrm{n}+1}=0$ (the calculation is in $\mathrm{Z}_{2}$ ) $\mathrm{r}_{1} \mathrm{P}_{2}+$ $r_{2} \mathrm{P}_{2}^{2}+\ldots+\mathrm{r}_{\mathrm{n}} \mathrm{P}_{2}^{\mathrm{n}}+\mathrm{r}_{\mathrm{n}+1} \mathrm{P}_{2}^{\mathrm{n}+1}=0$ (the calculation is in $\mathrm{Z}_{3}$ ) The check symbol $a_{n+1}=q_{n+1} * 9+r_{n+1}$.

## Initiation for the Contract-ID

$P_{1}$ is initial set to the $2 \times 2$ binary matrix $(0$
$\left.\begin{array}{l}1 \\ 1\end{array}\right)$ and $P_{2}$ to the $2 \times 2$ ternary matrix $\left(\begin{array}{lll}0 & & 1 \\ & & 1\end{array}\right)$.
11
The calculations in the two check equations are over $Z_{2}$ and $Z_{3}$, respectively., i.e., in the check equation which employs $P_{1}$, the calculation is over $Z_{2}$; and in the check equation whichemploys $P_{2}$, the calculation is over $Z_{3}$.

For the Contract-ID, the number of digits is $n=14$, and $n+1=15$ is the check digit.

## Setup for the Contract-ID

The binary and ternary matrixes to be used for the Contract-ID check digit calculation:

$$
\begin{aligned}
P & =\left(\begin{array}{ll}
0 & 1
\end{array}\right) \text { and } \\
1 & =\left(\begin{array}{ll}
1 & 1 \\
0 & 1 \\
1 & 2
\end{array}\right)
\end{aligned}
$$

The exponents of $P_{1}$ over $\mathrm{Z}_{2}$ :

$$
\begin{aligned}
P_{1} & =P_{1}^{4}=P_{1}^{7}=P_{1}^{10}=P_{1}^{13}=\left(\begin{array}{lll}
0 & 1 \\
P^{2} & =P_{1}^{5}=P_{1}^{8}=P_{1}^{11}=P_{1}^{14}=\left(\begin{array}{lll}
1 & 1 & 1
\end{array}\right) \\
& 1 & 0
\end{array}\right)
\end{aligned}
$$

$$
\underset{{ }_{1}}{P_{3}^{3}}=P_{1}{ }^{6}=P_{1}{ }^{9}=P_{1}{ }^{12}=P_{1} 15=\left(\begin{array}{lll}
1 & & 0 \\
& 0 & 1
\end{array}\right)
$$

The exponents of $P_{2}$ over $Z_{3}$ :

$$
\begin{aligned}
& P=P^{9}=\left(\begin{array}{ll}
0 & 1 \\
1 & 2
\end{array}\right) \\
& \mathrm{P}_{2}^{2}=\stackrel{2}{\mathrm{P}^{10}}{ }_{2}^{10}=\left(\begin{array}{ll}
1 & 2 \\
1 & 2 \\
& 2
\end{array}\right) \\
& P_{2}{ }^{3}=P_{2}^{11}=\left(\begin{array}{cc}
2^{2} & 2 \\
2 & 0
\end{array}\right) \\
& P_{2}{ }^{4}=P_{2}^{12}=\left(\begin{array}{cc}
2^{2} & 0 \\
0 & 2
\end{array}\right) \\
& \mathrm{P}_{2}^{5}=\mathrm{P}_{2}^{13}=\left(\begin{array}{cc}
0 & 2 \\
2 & 1
\end{array}\right) \\
& P_{2}{ }^{6}=P_{2}^{14}=\left(\begin{array}{ll}
2^{2} & 1 \\
1 & 1
\end{array}\right) \\
& P^{7}=P^{15}=\left(\begin{array}{ll}
1 & 1 \\
1 & 1
\end{array}\right) \\
& \mathrm{P}^{2}{ }_{2}^{8}=\left(\begin{array}{ll}
1^{2} & 0 \\
0 & 1
\end{array}\right)^{1}
\end{aligned}
$$

Lookup Tables
The correlation between the used alphabet in the Contract-ID and the values of $q_{n}$ and $r_{n}$ canbe implemented by using the following lookup tables:

| Alpha to $q_{1}$ |  |
| :---: | :---: |
| char | q1 |
| 0 | 0 |
| 1 | 0 |
| 2 | 0 |
| 3 | 0 |
| 4 | 0 |
| 5 | 0 |
| 6 | 0 |
| 7 | 0 |
| 8 | 0 |
| 9 | 0 |
| A | 0 |
| B | 0 |
| C | 0 |
| D | 0 |
| E | 0 |
| F | 0 |
| G | 0 |
| H | 0 |
| I | 1 |
| J | 1 |
| K | 1 |
| L | 1 |
| M | 1 |
| N | 1 |
| 0 | 1 |
| P | 1 |
| Q | 1 |
| R | 1 |
| S | 1 |
| T | 1 |
| U | 1 |
| V | 1 |
| W | 1 |
| X | 1 |
| Y | 1 |
| Z | 1 |


| Alpha to $q_{2}$ |  |
| :---: | :---: |
| char | q2 |
| 0 | 0 |
| 1 | 0 |
| 2 | 0 |
| 3 | 0 |
| 4 | 0 |
| 5 | 0 |
| 6 | 0 |
| 7 | 0 |
| 8 | 0 |
| 9 | 1 |
| A | 1 |
| B | 1 |
| C | 1 |
| D | 1 |
| E | 1 |
| F | 1 |
| G | 1 |
| H | 1 |
| I | 0 |
| J | 0 |
| K | 0 |
| L | 0 |
| M | 0 |
| N | 0 |
| 0 | 0 |
| P | 0 |
| Q | 0 |
| R | 1 |
| S | 1 |
| T | 1 |
| U | 1 |
| V | 1 |
| W | 1 |
| X | 1 |
| Y | 1 |
| Z | 1 |


| Alpha to $r_{1}$ |  |
| :---: | :---: |
| char | r1 |
| 0 | 0 |
| 1 | 0 |
| 2 | 0 |
| 3 | 1 |
| 4 | 1 |
| 5 | 1 |
| 6 | 2 |
| 7 | 2 |
| 8 | 2 |
| 9 | 0 |
| A | 0 |
| B | 0 |
| C | 1 |
| D | 1 |
| E | 1 |
| F | 2 |
| G | 2 |
| H | 2 |
| I | 0 |
| J | 0 |
| K | 0 |
| L | 1 |
| M | 1 |
| N | 1 |
| 0 | 2 |
| P | 2 |
| Q | 2 |
| R | 0 |
| S | 0 |
| T | 0 |
| U | 1 |
| V | 1 |
| W | 1 |
| X | 2 |
| Y | 2 |
| Z | 2 |


| Alpha to $r_{2}$ |  |
| :---: | :---: |
| char | r2 |
| 0 | 0 |
| 1 | 1 |
| 2 | 2 |
| 3 | 0 |
| 4 | 1 |
| 5 | 2 |
| 6 | 0 |
| 7 | 1 |
| 8 | 2 |
| 9 | 0 |
| A | 1 |
| B | 2 |
| C | 0 |
| D | 1 |
| E | 2 |
| F | 0 |
| G | 1 |
| H | 2 |
| 1 | 0 |
| J | 1 |
| K | 2 |
| L | 0 |
| M | 1 |
| N | 2 |
| 0 | 0 |
| P | 1 |
| Q | 2 |
| R | 0 |
| S | 1 |
| T | 2 |
| U | 0 |
| V | 1 |
| W | 2 |
| X | 0 |
| Y | 1 |
| Z | 2 |


| Reverse lookup |  |
| :---: | :---: |
| result | Check digit |
| 0 | 0 |
| 16 | 1 |
| 32 | 2 |
| 4 | 3 |
| 20 | 4 |
| 36 | 5 |
| 8 | 6 |
| 24 | 7 |
| 40 | 8 |
| 2 | 9 |
| 18 | A |
| 34 | B |
| 6 | C |
| 22 | D |
| 38 | E |
| 10 | F |
| 26 | G |
| 42 | H |
| 1 | I |
| 17 | J |
| 33 | K |
| 5 | L |
| 21 | M |
| 37 | N |
| 9 | 0 |
| 25 | P |
| 41 | Q |
| 3 | R |
| 19 | S |
| 35 | T |
| 7 | U |
| 23 | V |
| 39 | W |
| 11 | X |
| 27 | Y |
| 43 | Z |

## Example

The calculation of the check digit for the single Contract-ID 'DE83DUIEN83QGZ' is shownin the next steps as an example. (With $\mathrm{n}=14$ digits.)

## STEP 1:

According to the lookup table, we get the following matrices for the digits of the givenContractID:

$$
\begin{aligned}
& D \rightarrow\left({ }^{q^{1}}\right)=\left(\begin{array}{ll}
0 & 1
\end{array}\right) \\
& E \rightarrow\left(q^{2}\right)=\left(\begin{array}{ll}
\mathrm{r}_{1} & 1 \\
& 1
\end{array}\right) \\
& 8 \rightarrow\left(\mathrm{q}^{3}\right)=\left(\begin{array}{ll}
\mathrm{r}_{2} & 2 \\
& 0
\end{array}\right) \\
& \left.3 \rightarrow \underset{r_{4}}{\left(q^{4}\right.}\right)=\left(\begin{array}{ll}
\mathrm{r}_{3} & 2 \\
1 & 0
\end{array}\right) \\
& D \rightarrow\left({ }^{5}\right)=\left(\begin{array}{ll}
0 & 1
\end{array}\right) \\
& U \rightarrow\binom{\mathrm{r}_{5}}{\mathrm{q}^{6}}=\left(\begin{array}{cc}
1 & 1 \\
1 & 1
\end{array}\right) \\
& \left.\mathrm{I} \rightarrow \underset{\mathrm{r}_{7}}{\left(\mathrm{q}^{7}\right.}\right)=\left(\begin{array}{ll}
1 & 0 \\
0 & 0
\end{array}\right) \\
& E \rightarrow\left(q^{8}\right)=\left(\begin{array}{ll}
0 & 1
\end{array}\right) \\
& N \rightarrow\binom{\mathrm{r}_{8}}{\mathrm{q}^{9}}=\left(\begin{array}{cc}
1 & 2 \\
& 0
\end{array}\right) \\
& 8 \rightarrow \underset{r_{10}}{\left({ }^{\mathrm{r}_{9}}\right.} \mathbf{r _ { 1 0 }}=\left(\begin{array}{cc}
1 & 2 \\
0 & 0 \\
2 & 2
\end{array}\right) \\
& 3 \rightarrow\binom{\mathrm{r}_{10}}{\mathrm{q}^{11}}=\left(\begin{array}{ll}
\mathrm{r}_{11}^{2} & 2 \\
0
\end{array}\right) \\
& Q \rightarrow\binom{\mathrm{r}_{11}}{\mathrm{q}^{12}}=\left(\begin{array}{cc}
1 & 0 \\
& 0
\end{array}\right) \\
& G \rightarrow\left(\mathrm{q}^{13}\right)=\left(\begin{array}{ll}
0^{2} & 2 \\
& 1
\end{array}\right) \\
& Z \rightarrow \underset{r_{14}}{\binom{\mathrm{r}_{13}}{\mathrm{r}_{14}}}=\left(\begin{array}{ll}
1^{2} & 1 \\
1 & 1 \\
2 & 2
\end{array}\right)
\end{aligned}
$$

Step 2.
Calculate the check digit $\left(\mathrm{q}^{15}\right) \underset{\mathrm{r}^{15}}{\text { using }}$ the following check equation:
Check equation 1) calculated over $\mathrm{Z}_{2}$ :

$$
\mathrm{q}_{1} \mathrm{P}_{1}+\mathrm{q}_{2} \mathrm{P}_{1}^{2}+\cdots+\mathrm{q}_{15} \mathrm{P}^{15}=0
$$

Check equation 2) calculated over $\mathrm{Z}_{3}$ :

$$
\mathrm{r}_{1} \mathrm{P}_{2}+\mathrm{r}_{2} \mathrm{P}_{2}^{2}+\cdots+\mathrm{r}_{15} \mathrm{P}^{15}=0
$$

The calculation gives us $\left({ }^{q^{15}}\right)=\left(\begin{array}{ll}0 & 1\end{array}\right)$ which corresponds to the check digit $D$.

## STEP 3.

To validate a given check digit the steps 1 and 2 are to be applied to the ID without the appended check digit. The calculated result is to be compared to the given check digit inplace.

## Implementation

Due to the fact that only basic mathematical operations ( + , ${ }^{*}$, lookup) on small numbers ( $\{0,1,2\}$ ) are required, the check digit calculation and evaluation can be implemented very efficiently. Even bitwise implementation is possible, if wished. Moreover, depending on thehardware and software used, the speed for mod 2 and mod 3 can be much faster than for
$\bmod 36$ or $\bmod 1271$. Finally, the same operations are applied for the calculation and evaluation which can lead to lower implementation efforts. The following considerations referto the computational efforts per calculation/evaluation step:

## STEP 1:

The lookup tables can be 'hardcoded' per alphanumeric character of the ID-String so that foreach character only 1 lookup is necessary where each lookup results in 4 numbers ( $\Rightarrow 14$ lookups • 4 values $=56$ values)

## STEP 2:

Only the 56 values of step 1 are used as input for the solving of the two check equations in step 2. In order to derive the solution vectors $q_{15}$ and $r_{15}$, the first 14 terms ( $x \quad{ }_{1} P_{m}^{1} \ldots x{ }_{14} P_{m}^{14}$,
with $x=\{q, r\}$ and $m=\{1,2\}$ ) are to be folded at first. Due to the chosen initiation of $P_{1}$ and $P_{2}$, there are only three distinct matrices for the first check equation and eight distinct matrices for the second check equation across the 14 exponents. Due to the distributive property of matric multiplication, the first 14 terms of each check equation can be aggregatedto the vector terms $t_{1}$ and $t_{2}$ of three and eight blocks respectively. This allows for a more efficient implementation than in the accompanying reference implementation where the 14 terms are considered independently (.xls-file). For example, the aggregation term for the firstcheck equation is:

$$
\begin{aligned}
& T_{1}=\left(q_{1}+q_{4}+q_{7}+q_{10}+q_{13}\right) P_{1}+\left(q_{2}+q_{5}+q_{8}+q_{11}+q_{14}\right) P^{2} \\
&+\left(q_{3}+q_{6}+q_{9}+q_{12}\right) P^{3}
\end{aligned}
$$

Folding this term within each block requires across all blocks 11 vector summations $(\Rightarrow 22$ summations). Multiplying a vector with a $2 \times 2$-matrice requires 4 multiplications and 2 summations. Since $P^{3}$ is the identity matrice, this multiplication has to be done only for the first two blocks ( $\Rightarrow 4$ summations and 8 multiplications). Considering the summing up of thethree blocks ( $\Rightarrow 6$ summations) eventually leads to a total effort of 32 summations and 8 multiplications for $t_{1}$. Correspondingly, $t_{2}$ consists of eight blocks resulting in seven vector summations ( $\Rightarrow 14$ summations). With $P^{8}$ being the identity matrice, only the first seven blocks have to be multiplied with the corresponding $2 \times 2$-matrices ( $\Rightarrow 14$ summations and 28 multiplications). Considering the summing up of the seven blocks ( $\Rightarrow 14$ summations) eventually leads to a total effort of 42 summations and 28 multiplications for $t_{2}$.

If the used programming language does not allow for native calculations in $Z_{2}$ and $Z_{3}$, the terms $t_{1}$ and $t_{2}$ can be calculated in Z or R (due to the distributive property of matrice operations). In this case the modulo-calculation needs to be applied after the folding on eachvalue of both vector terms ( $\Rightarrow 4$ modulo operations).

Finally, solving of the resulting two aggregated check equations $\left(t_{m}+x_{15} * P^{15}=0\right.$, with $x=\{q, r\}$ and $m=\{1,2\}$ ) requires two comparisons for both entries of vector $q_{15}$ as well asthree comparisons for both entries of vector $r_{15}(\Rightarrow 10$ comparisons in total; alternatively, a conventional resolving could also be done very efficiently: Due to $P^{15}$ and $P_{1}^{15}$ being ${ }_{2}$
constant, vector $q$ could be caculated with two summations and one 'mod 2'-operation. Vector $r$ could be calculated with two summations and two 'mod 3'-operations; for the transformation of the linear equations cf. also the comments in the accompanying .xls-file).

## STEP 3:

Cf. Step 1 and 2.

## CONCLUSION

All in all, each check digit can be either calculated or evaluated with the following basic mathematical operations on very small numbers (for $Z_{2}$ in $\{0,1\}$ and for $Z_{3}$ in $\{0,1,2\}$ ):

- 14 table lookups
- 36 multiplications
- 74 summations
- 4 modulo operations
- 10 comparisons

Although this algorithm may require a few more basic mathematical operations than modulo check digits, all hard- and software is easily able to conduct these calculations very fast - andyou catch all the most frequent error types.

The above descriptions originate from email-exchange with Yanling Chen (Department ofTelematics at NTNU, Trondheim, Norway) between 12/2012-01/2013.

The algorithm is based on:

> Chen, Y., Niemenmaa, M., \& Vinck, A. (2013). A check digit system over a group ofarbitrary order.
> 2013 8th International Conference on Communications and Networking in China (CHINACOM) (pp. 897-902). IEEE.
> http://ieeexplore.ieee.org/xpl/articleDetails.jsp?arnumber=6694722

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